Summary

This report addresses the mathematical knowledge and skills required for civil engineering students in their introductory courses. One of the key elements identified throughout this document has been the need to integrate mathematics and engineering curricula so that students apply their mathematical skills soon after the material is taught. In order to accomplish this goal, it is necessary to maintain interactions between mathematics and engineering faculty and it may be advisable for introductory mathematics courses to be provided over a three or four year time period rather than a two year period.

In addition, it is essential for mathematics curriculum to focus on developing problem-solving skills. This can be accomplished by adjusting the balance of course material to more applied and numerical solution techniques and less advanced analytical techniques, integrating more technology into the curriculum, and coordinating the mathematics and engineering curricula. Our goal is not to dilute the introductory mathematics curriculum but rather to provide more in-depth understanding and applications of basic concepts prior to introducing more complex topics. The net effect of this approach will be that students gain a greater appreciation of mathematics and will be encouraged to pursue these more complex concepts in advanced courses.

Narrative

Introduction and Background

Civil Engineering is a field of broad scope that ranges from the design and construction of structures, roadways and pollution control processes to the management of our natural and engineered resources. The breadth of the field has led to division into a number of subdisciplines including structural engineering, geotechnical engineering, transportation engineering, environmental engineering, materials, construction management, and water resources engineering. Civil engineering draws on a number of science disciplines including chemistry, physics, ecology, geology, microbiology, material science, economics and mathematics, and statistics.

The level of mathematical and statistical skills required of civil engineering professionals varies with the type of work and the subdiscipline. Many of the technical mathematics skills required for our students are needed primarily to understand the concepts introduced in their engineering courses. The majority of our graduates employ very little of the technical mathematics skills that they learned in college on a day
to day basis; however, they rely heavily on their problem solving skills. Very few of our students graduating with Bachelor of Science degrees code new software or solve problems analytically, yet they rely on very sophisticated software and tools that use all levels of advanced mathematics. Still, a significant fraction of our graduates utilize advanced numerical modeling and statistical analyses to evaluate data and predict performance. It is at the graduate level where advanced mathematics is used most heavily. In addition, one of the most significant trends in the past two decades has been the reliance on spreadsheets and commercially available software packages for solving many routine mathematical problems.

This report summarizes the panel’s evaluation of the curriculum needs for introductory mathematics courses for civil engineering students. Our recommendations focus on several key themes that we believe are essential for improving the quality of mathematics and engineering education:

• A strong emphasis should be given to developing problem-solving skills across the curriculum.

• The introduction of engineering content should start early in the curriculum and the timing of mathematics curriculum content should be closely linked to its use in engineering courses.

• Introductory mathematics content should focus on developing a sound understanding of key fundamental concepts and their relevance to applied problems.

• A stronger emphasis should be placed on numerical solution techniques (e.g., root finding, interpolation, curve fitting, numerical differentiation and integration).

• Curriculum reform requires interdisciplinary coordination between provider and end-user departments.

These themes are re-iterated and expounded upon as we address several key areas of concern for mathematics curriculum. These areas include: understanding and content, technology, instructional interconnections, and instructional techniques.

**Understanding and Content**

There are a number of early conceptual mathematical principles that are required for civil engineering students. The range of topics is rather broad and covers the major concepts in algebra, trigonometry, logarithms, graphical analysis, data transformation, systems of equations, vectors, series, matrix algebra, integration, differentiation, basic differential equations, probability, statistics and optimization. A complete list of these topics is included in the Appendix.

It may be beneficial to evaluate the order of topics in order to arrange for them to coincide more closely with their use in engineering and basic science courses, especially physics. For example, basic numerical methods and matrix algebra are often stressed very early in the engineering curriculum. In contrast, multivariable calculus is not used until much later. If students focus on gaining a better understanding of basic principles and have a chance to apply them in their engineering classes before they are introduced to more complex topics, they may gain a greater appreciation of mathematics and maintain their enthusiasm for learning new concepts.

The major difference between the needs of current and past civil engineers relates to the techniques available for solving complex problems. Whereas in the past it was necessary to provide in-depth treatment of many different analytical techniques for solving problems in each of these areas, the advent of calculators, computers, and user-friendly software packages that replaced the need for broad instruction in numerous solution techniques that are now rarely used in practice. Rather, introductory mathematics instruction should concentrate on teaching the major concepts of each of these areas, their physical meaning and their application to solving realistic problems.

In addition, it is extremely important that students develop the ability to recognize the application of these concepts to applied problems. For example, we believe that it is important for students to understand that integration can be used to determine the area under a curve, and that differentiation can be used to determine the slope of a curve at a particular location. Furthermore, when students are faced with a prob-
Problem in which they must determine the area under a curve they should be able to recognize that they must integrate and that there are both analytical and numerical tools available to accomplish this task. In contrast, it is less important for students to learn the multitude of analytical techniques for integrating, especially techniques that apply to complex problems that are more likely to be solved numerically.

As a result, students should learn both analytical and numerical solution techniques in their mathematics courses. They should understand the reasons for selecting a particular technique, develop an understanding of the range of applicability of the technique, acquire familiarity with the mechanics of the solution technique, and understand the limitations of the technique.

In order to acquire this level of understanding, analytical solution techniques should be taught in conjunction with numerical techniques. Analytical solutions should generally be introduced using relatively simple examples so that students can develop a sound understanding of the concepts. Numerical techniques for solving similar problems should also be introduced so that students can make the connection between the analytical and numerical solutions. For more complex problems, numerical solutions should be emphasized; however, analytical solutions (perhaps under constrained conditions) should be re-emphasized to stress the need for validating numerical solutions. With this approach students can develop a sound understanding of the fundamental concepts presented and learn solution techniques that can be applied to solving realistic problems.

The development of problem-solving skills is one of the primary goals of the civil engineering curriculum. Problem solving involves five basic components: recognize and define the problem; formulate the model and identify variables, knowns and unknowns; select an appropriate solution technique and develop appropriate equations; apply the solution technique (solve the problem); and validate the solution. Solution validation is one of the most important steps in this process and includes interpreting the solution, identifying its limitations, and assessing its reasonableness using appropriate approximate solutions or common sense.

As an example of this approach, consider the following problem. Design a cylindrical aluminum can that can hold 400 mL and that minimizes the quantity of aluminum used. The solution for this problem can be described as follows:

<table>
<thead>
<tr>
<th>Problem Solving Component</th>
<th>Solution</th>
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<tr>
<td>1. Recognize and define the problem</td>
<td>Minimize the surface area of the can</td>
</tr>
</tbody>
</table>
| 2. Formulate the model and identify variables, knowns and unknowns | Surface Area = \(2\pi R^2 + 2\pi Rh\)  
Volume = \(\pi R^2 h = 400\) cm\(^3\)  
Radius \(R\) and height \(h\) are unknown |
| 3. Solution Technique | When the derivative of the surface area with respect to the radius equals 0, the surface area is a minimum or maximum. Differentiate analytically to find \(dA/dR\). |
| 4. Solve the equation | \(dA/dR = 4\pi R - 800/R^2 = 0\)  
\(R = 3.99\) cm and \(h = 7.98\) cm |
| 5. Validate the solution | Is the solution a minimum? Evaluate the 2nd derivative: \(d^2A/dR^2 = 4\pi + 1600/R^3\) which is positive for \(R\) greater than zero. Or test the surface area equation using values of \(R\) that are greater and smaller than 3.99, and determine that \(R = 3.99\) provides the minimum surface area. |

We believe that it is essential for students to be exposed to problem solving techniques in their mathematics courses as well as in their engineering courses. It is not necessary that the problems used in mathematics
courses be related to engineering, but it is essential that students gain continual exposure to the tools required to solve engineering problems and to develop the ability to apply their mathematical skills to applied problems.

Technology

Technology (i.e., computer software, graphing calculators) must be a major component of mathematics curriculum. However, it is important that technology be applied in an appropriate manner and at the appropriate time in the curriculum. Many students entering college have been exposed to technology in their classrooms for several years and in their day-to-day lives. One of the keys to maintaining their enthusiasm for mathematics is to show them how technology can be used to solve real problems and to help them to realize that this same technology will be used in their engineering classes and in their careers.

The incorporation of technology into the mathematics curriculum means that some topics must be sacrificed. However, one of our recommendations for changing the mathematics curriculum is to place less emphasis on learning complex analytical solution techniques and to spend more time emphasizing the application of numerical solutions and computer based analytical techniques, problem analysis, and problem solving skills. By foregoing these more complicated analytical techniques two goals can be accomplished; more emphasis can be placed on gaining a deeper understanding of concepts through the application of both analytical and numerical solutions, and students gain an appreciation for the benefits of technology in mathematics and engineering.

Thus, while students need to apply technology to solve complex problems, the technology must be used in a manner that promotes understanding of mathematical concepts and their applications. In many instances, students learn how to apply technology (i.e., they know how to make software provide answers) without understanding the solution approach. This trap must be avoided.

Several basic technology skills should be taught in introductory courses to complement the theoretical and analytical treatment of mathematics topics. These address the areas of graphical techniques, algorithm development, symbolic manipulation and equation solving, spreadsheet operations, and basic computer programming. Students should be able to:

- Graph analytical functions using a graphing calculator or computer.
  Many students are visual learners and the ability to graph functions is an essential part of grasping new concepts and evaluating the effects of different parameters on the shape of a function.
- Use spreadsheet and data management software such as Microsoft Excel and Microsoft Access during their introductory courses.
  The level of familiarity should include the use of macros in spreadsheet programs and the capability to perform statistical analyses and equation solver routines.
- Use equation solvers and programs capable of symbolic manipulation such as MathCad.
- Implement a computer language such as Visual Basic.
  The goal in learning a computer language is to develop an understanding of programming techniques. The depth of knowledge need not be substantial and the selection of the specific language is less important than the development of detailed logic and flow charts. Students who go beyond an undergraduate degree will need programming skills to a much greater extent, but these skills can be acquired in more advanced courses. In addition, specific software packages are often taught within specialty courses.

With all of these tools it is essential that they be introduced to students at the appropriate time; after the student acquires a conceptual understanding of the technique but before the tedium of solving problems by hand frustrates them. In addition, the use of these tools should be coordinated between mathematics and engineering. The need for cooperation between faculty in these disciplines cannot be underestimated.
Ideally, implementation of this cooperative approach will take place over three to four years of students’ undergraduate career. Indeed, in order for the mathematics content to link to the engineering curriculum content, it may be appropriate for students to take introductory mathematics courses in their freshman, sophomore and junior years.

### Instructional Techniques

A number of instructional methods have proven effective for developing mathematical comprehension. The most important of these is the use of hands-on, active learning techniques in the classroom.

Of equal importance is the need to make students understand the utility of the material they are being taught. Students need to understand and appreciate the need for their courses. Many engineering students leave their mathematics courses thinking that the material will never be used in their engineering courses. It is essential that mathematics courses have some future value in their engineering courses. The mathematics portion of a student’s curriculum should not be simply something “to get through.” This means that engineering and mathematics faculty must coordinate their curriculum. Mathematics faculty must teach methods that are applicable to current engineering practice, and engineering faculty must employ these methods in their curriculum within a reasonable time period after students learn the techniques.

Finally, it is necessary that institutions and national organizations provide resources for faculty development to implement curriculum reform and new teaching methods. Every Ph.D. program in mathematics and engineering should have a formal requirement for teacher development. Departments should provide incentives for current faculty to become involved in curriculum and pedagogical reform.

### Instructional Interactions

Engineering pedagogy has advanced dramatically over the past decade. There has been a significant increase in team interaction. Our courses rely more heavily on the use of technology for instruction and as part of the curriculum. Active and cooperative learning is stressed in many of our classes. Many professors have reduced the number of topics in their classes in order to provide greater understanding and retention. There is a stronger emphasis on teaching problem-solving skills and providing more open-ended problems that closely simulate the types of problems encountered in practice.

While many of these topics have also been addressed in the mathematics curriculum, there have been very few examples where engineering faculty have been involved or even informed as to the types of changes that have been implemented. One of the most significant conclusions of this report is that there is a need for much greater interaction between engineering and mathematics faculty regarding curriculum reform. This interaction needs to occur formally at both the national and institutional levels. Existing forums such as the American Society of Engineering Education and the Mathematics Association of America can be used to initiate interactions at the national level. It is incumbent upon each university involved in mathematics and engineering reform to coordinate curriculum changes to ensure success. In addition, more studies are required that track students and assess the outcome of students who have been involved in curriculum reform.
WORKSHOP PARTICIPANTS

Robert Henry, Associate Professor and Associate Dean, Engineering and Physical Sciences, University of New Hampshire
Marc Hoit, Professor and Associate Dean, Civil Engineering, University of Florida
Lynn Katz, Associate Professor, Civil Engineering, University of Texas at Austin (coordinator)
Saleh Keshawarz, Associate Professor and Chair, Civil and Environmental Engineering, University of Hartford
Fred Hart, Professor and Head, Civil and Environmental Engineering, Worcester Polytechnic University
Matt Ohland, Assistant Professor, General Engineering, Clemson University.
Ben Sill, Alumni Professor and Director of General Engineering, Civil Engineering, Clemson University
David Thompson, Associate Professor, Civil Engineering, Texas Tech University

Mathematics Participants

Susan Ganter, Associate Professor, Mathematical Sciences, Clemson University
William McCallum, Professor, Mathematics, University of Arizona
APPENDIX: Mathematics Topics
Relevant to Civil Engineering

Basics
- Algebra
- Trigonometry
- Logarithms
- Graphical Analysis of Functions
- Data Transformation (e.g., $y = axb \Rightarrow \log y = \log a + b \log x$)
- Systems of Equations
- Iterative Solutions

Vectors
- Addition
- Dot Products
- 3-D Coordinate Systems
- 3-D Visualization
- Coordinate Transformation
- Translation, Rotation

Matrix Algebra
- Linear Systems
- Transformations
- Determinants (3x3)

Series
- Taylor Series (Approximations)

Differentiation
- Definition of a Derivative
- Derivative as a Slope of a Curve
- Min, Max
- Derivatives of Polynomials, Trigonometric Functions and Exponentials
- Chain Rule
- Root Finding

Integration
- Definite vs. Indefinite Integrals
- Integration as the Area under a Curve
- Basic Integration Techniques
- Multiple Integrals (Two)
- Areas, Volumes, Centroids, Moments
- Numerical Integration

Differential Equations
- First Order ODE – Linear, Constant Coefficient
- Second Order ODE – Single Variable
- Initial and Boundary Conditions
- Laplace Transformations
Partial Differential Equations (Intro. – Single Spatial Variable)
Numerical Methods – Euler’s, Runga Kutta

**Probability/Statistics/Optimization**
Regression, Curve Fitting
Distributions — Triangular, Normal
Return Intervals
Linear Programming — Simplex Method